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Final Exam

1.

1) D
2) B
3) D
4) D
5) B
6) B
7) A
8) A
9) B
10) D
11) B
12) C

2.

a)

p	q	t	$p \to t$	$q \to t$	$(p \to r) \land (q \to r)$	$p \lor q$	$(p \lor q) \to r$
T	T	T	T	T	T	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	T	T	T	F	T
T	T	F	F	F	F	T	F
T	F	F	F	T	F	T	F
F	T	F	T	F	F	T	F
F	F	F	T	T	T	F	T

b) $(p \to t) \land (q \to t) \equiv (\neg p \lor t) \land (\neg q \lor t)$ by Representation of If-Then as or
$\equiv (\neg p \land \neg q) \lor t$ by Distributive laws
$\equiv \neg (p \lor q) \lor t$ by De Morgan's law
$\equiv (p \lor q) \to t$ by Representation of If-Then as Or

3.

a) A is a knight and B is a knave
b) There is one knave

4.

a) If n^2= a factor of 7, then n = a factor of seven
The proof is that for any given n^2 that is divisible by 7, n is a factor of n^2, and since 7 is a factor of n^2, 7 must be a factor of n as well
b) Assume that the square root of 7 is rational, this means that the square root of 7 can be represented as a/b, which can be phrased as $a = b\sqrt{7}$. We can then square both sides to produce $a^2=7b^2$. This means that a is divisible by 7. We can then represent a as a = 7x, since

a is divisible by 7. We can then substitute this into $a^2=7b^2$ we obtain $49x^2=7b^2$, which equals $7x^2=b^2$, which means that b^2 is divisible by 7. And by our proof of this in (4.a) we know this means b is divisible by 7 as well. This supposes that a and b have a common factor, but this is a contradiction since a and b are co-prime which means that the square root of 7 is irrational

c) Assume that the $\log_2(7)$ is rational. Then this can be represented as $\log_2(7) = a/b$. This means that, by the definition of a logarithm, $2^{(a/b)} = 7$. This can be rewritten as $2^a = 7^b$. The left-hand side will thus equal an even number while the right-hand number will equal an odd number. Thus $\log_2(7)$ cannot be rational

5.

a) This can be represented as:

$$\sum_{i=1}^{n} i^3 = \left(\frac{n^2+n}{2}\right)^2$$

Base case: when n = 1

LHS = $1^3 = 1$

RHS = $\left(\frac{n^4+n^3}{2}\right)^2 = 1$

Then when n = k+1

$$\sum_{i=1}^{k+1} i^3 + (k+1)^3$$

$$\left(\frac{k^2+k}{2}\right)^2 + (k+1)^3$$

$$\frac{(k^2+k)^2}{4} + \frac{(k+1)^3}{1}$$

$$\frac{(k^2+k)^2}{4} + \frac{4(k+1)^3}{4}$$

$$\frac{(k^2+k)^2 + 4(k+1)^3}{4}$$

$$\frac{k^4+2k^3+k^2+4k^3+12k^2+12k+4}{4}$$

$$\frac{k^4+6k^3+13k^2+12k+4}{4}$$

$$\frac{(k+1)^2(k+2)^2}{4}$$

$$\left(\frac{(k+1)(k+2)}{2}\right)^2$$

Thusly

$$\left(\frac{n^2+n}{2}\right)^2 = \left(\frac{(k+1)(k+2)}{2}\right)^2$$

6.

a) This is false for the sets $A = \{1, 2\}$ and $B = \{2, 3\}$
LHS = All elements of U except for 1, 2, 3
RHS = All elements except 1, 2 + all elements except 2, 3
= All elements except 2

b) This is false for sets $A = \{1, 2, 3, 4\}$ and $B = \{1, 2\}$ and $C = \{2, 3\}$
$A\text{-}B = \{3, 4\}$, $A\text{-}C = \{1, 4\}$
$\{3, 4\} \cap \{1, 4\} = \{4\}$